Energy based Control Barrier functions for Robotic Manipulators with Large Safety Constraints

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***Abstract*— In this paper, we show how to realize robust safety-critical control laws for robotic manipulators with a large number of inequality constraints (>100). In particular, we use control barrier functions (CBFs) formulated via the kinetic energy terms to represent constraints like joint position and velocity limits, both in configuration and task space. By using the kinetic energy terms, we can realize model-free constraints in a quadratic program (QP), which can be solved in real-time, thereby demonstrating fast computation time despite the presence of large constraints. We will consider two types of CBFs, the reciprocal and the zeroing type, and integrate with Control Lyapunov Function (CLF) based constraints to yield a multi-objective QP. Further, we will provide feasibility and continuity guarantees, thereby yielding a continuous, robust and a safe control law for a broad class of robotic systems. Towards the end, we will demonstrate two types of QP formulations in a 6-DOF manipulator; one uses 109 constraints through the reciprocal type, and the other uses 61 constraints using the zeroing type of CBFs.**

I. Introduction

With an increasing number of robots being utilized in industries, safety has become an integral part. Thus, to prevent any accident in such environments, especially when human operators are working in the vicinity, it is imperative to incorporate safety at all layers. Accordingly, safety-critical control laws have been extensively explored by researchers in the recent years where the goal is to ensure that a system never enters an unsafe region [1]–[5]. Using control barrier functions (CBFs) we can synthesize controllers for nonlinear dynamical systems by providing formal safety guarantees [6], [7]. Several robotic applications are time sensitive and require that the safety guarantees be provided in real-time. In case of CBFs the constraints are expressed in the form of quadratic programming problems (QPs), which can easily be solved using efficient off-the-shelf solvers.

Although control barrier functions introduced in [6] are a popular choice for enforcing safety constraints, they are heavily model-based and hence lack robustness properties. Moreover, the traditional CBFs are not designed for incorporating large numbers of constraints and have feasibility and continuity issues which prevent their usage in practical applications. Recent work on energy based CBFs [8], [9], and even the more recent work on model-free CBFs [10] aimed to resolve the issue of handling model uncertainty, but not particularly on the issue of feasibility and continuity when a large number of constraints are considered [11], [12].

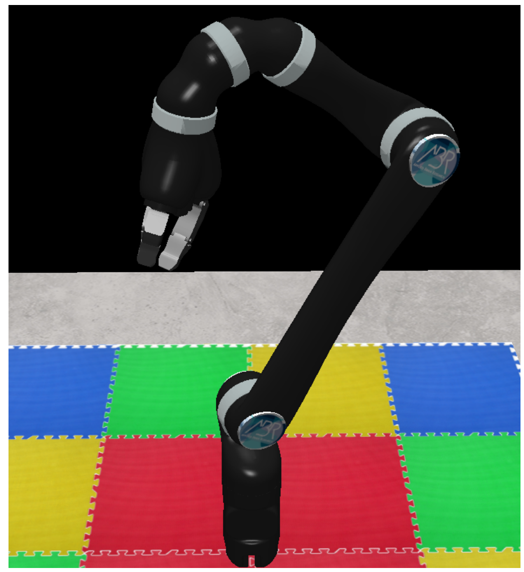

Fig. 1: 6 DoF robot manipulator (Mujoco based model), where we incorporate a large number of safety constraints (>100). We propose a QP based model-free control framework with guaranteed feasibility and continuity properties.

In this paper, we aim to resolve this issue of incorporating large inequality constraints and yet ensure feasibility, continuity and safety of the resulting QP formulations. In particular, we use a specific form of constraints obtained from the energy based CBFs [8], [9]. We provide QP formulations for both types of CBFs: reciprocal and zeroing control barrier functions, and in both joint and task spaces. In [9], zeroing type was shown for only position based constraints; here we extend it for velocity based constraints, and also show how it can be used in conjunction with other constraints. We show that, with torque limits permitting, the formulation always yields a feasible and a Lipschitz continuous control law. We implement the proposed control in a 6 DOF arm platform and demonstrate safety with >100 constraints. We also combine tracking using model-free Control Lyapunov Function (CLF) based constraints. Simulation results show that the proposed formulation is very effective for many practical scenarios.

The paper is organized as follows. The notations, robot dynamics and control of robot in task space is described in Section II. In Section III, the zeroing and reciprocal control barrier functions (CBFs) are defined along with their safety

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properties. Further, the energy based CBFs for position and velocity based constraints are discussed. In Section IV, the barrier functions for task space are introduced. Section V provides the unification of safety and stability by incorporating Control Lyapunov Functions (CLFs) with CBFs. We also provide our main results here. Section VI provides the simulation results for two types of QP formulations on a 6 DOF manipulator with position and velocity constraints enforced in task space and joint space. The manipulator stabilizes to a desired position and orientation while satisfying the safety constraints. Finally, Section VII concludes the paper.

II. Robot Dynamics and Task Space Control

In this section, we will first describe the robot dynamics and the task space control of the robot. In particular, we will describe Euler-Lagrangian formulation of the robot, and the required control law to make the end effector reach a desired configuration in the Cartesian space.

A. Notation.

A continuous function $\alpha : [0, a) \to [0, \infty)$ for some $a > 0$ is said to belong to *class* $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. Here, a is allowed to be $+\infty$. A continuous function $\alpha : [0, \infty) \to [0, \infty)$ is said to belong to *class* $\mathcal{K}_\infty$ if it is strictly increasing, $\alpha(0) = 0$, and $\alpha(r) \to \infty$ as $r \to \infty$. A continuous function $\alpha : (-b, c) \to (-\infty, \infty)$ is said to belong to extended class $\mathcal{K}$ for some $b > 0$, $c > 0$ if it is strictly increasing and $\alpha(0) = 0$ (see [11, Definition 1]). Here again, b, c are allowed to be $+\infty$. If we want to indicate the domains, we will denote class $\mathcal{K}$ and extended class $\mathcal{K}$ functions as $\mathcal{K}_{[0,a)}$, $\mathcal{K}_{(-b,c)}$ respectively. Given the state x, we denote its Euclidean norm as $|x|$.

B. Robot Dynamics

A fully actuated robotic manipulator with configuration manifold $\mathbb{Q}$ can be modeled by Euler-Lagrangian equations as follows:

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u, \tag{1}$$

where $q \in \mathbb{Q}$ denotes the configuration of the robot, $\dot{q} \in T_q\mathbb{Q}$ represents the rate of change of the configuration q, $D(q) \in \mathbb{R}^{n\times n}$ is the positive definite inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{n\times n}$ is the Coriolis-centrifugal matrix, $G(q) \in \mathbb{R}^n$ is the gravity vector, and $u \in \mathbb{U} = \mathbb{R}^n$ is the control input provided to the actuators. The above dynamics in (1) can also be written in control-affine state-space form as

$$\dot{x} = f(x) + g(x)u, \tag{2}$$

where f, g are the appropriate vector functions, and $x := (q, \dot{q}) \in T\mathbb{Q}$, denotes the vector constituting the state of the robotic manipulator, where $T\mathbb{Q}$ is the tangent bundle of $\mathbb{Q}$. In this paper, we employ a 6 DOF robotic manipulator (Kinova Jaco 2, see Fig. 1) for implementation of the proposed methodology.

The following properties hold true for a class of commonly studied mechanical systems including but not limited to serial manipulators (see [13]).

Property 1. *D is symmetric positive definite, and $\dot{D} - 2C$ is skew-symmetric $\forall q$ ([14], Lemma 4.2). In addition, there exist positive constants c_l, c_u, such that for any $(q, \dot{q}) \in T\mathbb{Q}$,*

- $c_l \leq \|D(q)\| \leq c_u$
- $c_l \leq \|D^{-1}(q)\| \leq c_u$
- $\|\dot{D}(q)\| \leq c_u|\dot{q}|$
- $\|C(q, \dot{q})\| \leq c_u|\dot{q}|$
- $|G(q)| \leq c_u$.

We have chosen the same constants c_u, c_l for ease of notations. Proof of Property 1 can be found in [13] - [15].

C. Task space control

The majority of tasks such as pick and place operation, hold and drop etc. involving robotic manipulators are defined in terms of the desired end effector position and orientation. In order to achieve the desired pose, the control design for such tasks can be performed by either transforming the trajectory in the task space to joint space and then designing a controller in the joint space (also known as joint space control) or it can be directly designed in the task space (also referred to as task space control). For controlling the end-effector pose, the dynamics of the manipulator should be defined in the task space, thus we reformulate the dynamics given in (1). We define the following:

$$e = \Phi(q), \dot{e} = J(q)\dot{q}. \tag{3}$$

where e denotes the state vector that includes end-effector position and orientation, and J is the Jacobian of the transformation $\Phi(q)$. We assume that J is invertible[1]. Accordingly, we have the dynamics as

$$D_e(q)\ddot{e} + C_e(q, \dot{q})\dot{e} + G_e(q) = J^{-T}u, \tag{4}$$

where $D_e(q) = J(q)^{-T}D(q)J(q)^{-1}$, $C_e(q, \dot{q}) = J(q)^{-T}C(q)J(q)^{-1} + J(q)^{-T}D(q)\frac{d}{dt}(J(q)^{-1})$ and $G_e(q) = J(q)^{-T}G(q)$. Property 1 for the task space dynamics can still be ensured locally (for a bounded set $\mathbb{N} \subset \mathbb{Q}$):

Property 2. *D_e is symmetric positive definite, and $\dot{D}_e - 2C_e$ is skew-symmetric. In addition, there exist $c_l, c_u > 0$ (possibly smaller, larger than previously determined c_l, c_u) such that for any $(q, \dot{q}) \in \bigcup_{q\in\mathbb{N}} T_q\mathbb{Q}$,*

1. $c_l \leq \|D_e(q)\| \leq c_u$
2. $c_l \leq \|D_e^{-1}(q)\| \leq c_u$
3. $\|\dot{D}_e(q)\| \leq c_u|\dot{e}|$
4. $\|C_e(q, \dot{q})\| \leq c_u|\dot{e}|$
5. $|G_e(q)| \leq c_u$.

This will be useful for allowing larger velocity variations. Proof of Property 2 is provided in [16, Appendix B]. The interested reader may also see [14, Chapter 4, Section 5.4] for more details. One of the advantages of task space control is that the inverse kinematics need not be calculated explicitly. The main goal of the task space control is to design a

[1] The end-effector space dimension is typically equal to the number of joints. In case if there are more joints, then the vector e would include the redundant joints, thereby ensuring non-singularity of J

feedback controller that allows execution of an end-effector motion $e(t) \in \mathbb{R}^n$ that tracks the desired end-effector motion $e_d(t) \in \mathbb{R}^n$ as closely as possible.

If the forces and moments acting on the end-effector are denoted by F, then using the Jacobian transpose the relation between the forces/torques acting on the joints of the manipulator (u) and the forces/torques acting on the end effector (F) can be given by

$$u_{ref} = J^T F. \tag{5}$$

The corresponding PD control law can be given by

$$u_{ref} = J^T (K_p(e_d - e) + K_d(\dot{e_d} - \dot{e})), \tag{6}$$

where K_p and K_d denote the proportional and derivative gains for the PD controller. In [17], it was shown how to incorporate PD control laws via control Lyapunov function based Quadratic Programs (CLF-QPs). The resulting formulation is not only model-free, but also allows us to include additional constraints like safety in the real-time controller. This will be described after describing model-free CBF-QPs.

III. Safety Critical Control

In this section, we will describe the generic formulation used for ensuring safety. Further, we will describe kinematic constraints associated with the Euler-Lagrangian formulation of the robot that are used. Typical constraints used in robotic systems are:

- Joint position based constraints:

$$\begin{aligned} h_{i,p,l}(q,\dot{q}) &:= q_i - q_{\min} \geq 0, \\ h_{i,p,u}(q,\dot{q}) &:= q_{\max} - q_i \geq 0. \end{aligned} \tag{7}$$

- Joint velocity based constraints:

$$\begin{aligned} h_{i,v,l}(q,\dot{q}) &:= \dot{q}_i - \dot{q}_{\min} \geq 0, \\ h_{i,v,u}(q,\dot{q}) &:= \dot{q}_{\max} - \dot{q}_i \geq 0. \end{aligned} \tag{8}$$

- End-effector position based constraints:

$$\begin{aligned} h_{i,e,l}(q,\dot{q}) &:= e_i(q) - e_{\min} \geq 0, \\ h_{i,e,u}(q,\dot{q}) &:= e_{\max} - e_i(q) \geq 0. \end{aligned} \tag{9}$$

- End-effector velocity based constraints:

$$\begin{aligned} h_{i,\dot{e},l}(q,\dot{q}) &:= \dot{e}_i(q,\dot{q}) - \dot{e}_{\min} \geq 0, \\ h_{i,\dot{e},u}(q,\dot{q}) &:= \dot{e}_{\max} - \dot{e}(q,\dot{q}) \geq 0. \end{aligned} \tag{10}$$

here i corresponds to the constraint on the i^{th} joint angle/end effector pose and q_{max}, e_{max} and q_{min}, e_{min} denote the maximum and minimum permissible joint angle/end effector pose respectively. These constraints can be represented in the form of sets, and the goal is to construct control laws that ensure forward invariance of these sets.

A. *Safety as forward invariance of a set*

The concept of control barrier functions (CBF) is derived from the concept of control Lyapunov function (CLF). CLFs ensure that the trajectory reaches a desired point in a set P, whereas the CBFs ensure that the trajectory remains inside a safe set. Consider the set $\mathcal{C} \subset T\mathbb{Q}$, defined as

$$\mathcal{C} = \{x \in T\mathbb{Q} : h(x) \geq 0\}, \tag{11}$$
$$\partial\mathcal{C} = \{x \in T\mathbb{Q} : h(x) = 0\}, \tag{12}$$
$$\mathrm{Int}(\mathcal{C}) = \{x \in T\mathbb{Q} : h(x) > 0\}, \tag{13}$$

where $h : T\mathbb{Q} \to \mathbb{R}$ is a continuously differentiable function. Note that h could be one of (7)-(10). It is assumed that $\mathrm{Int}(\mathcal{C})$ is non-empty and $\mathcal{C}$ has no isolated points, i.e., $\mathrm{Int}(\mathcal{C}) \neq \emptyset$, and $\overline{\mathrm{Int}(\mathcal{C})} = \mathcal{C}$.

We are interested in an optimal control law, $k : T\mathbb{Q} \to \mathbb{R}^n$, that can be specified that will ensure safety of $\mathcal{C}$ (or of $\mathrm{Int}(\mathcal{C})$), i.e., ensure that $h(x(t)) > 0 \ \forall t \geq 0, x(0) \in \mathrm{Int}(\mathcal{C})$, which is obtained via control barrier functions (CBFs). Specifically, we will study two types: reciprocal control barrier functions (RCBFs) and zeroing control barrier functions (ZCBFs).

B. *Control barrier functions*

In this subsection we will formally describe two classes of *control barrier functions* for robotic systems, and the associated quadratic programming (QP) formulation that ensures forward invariance of a safe set.

Definition 1. *Given a set* $\mathcal{C} \subset T\mathbb{Q}$ *defined by* (11)-(13) *for a continuously differentiable function* $h : T\mathbb{Q} \to \mathbb{R}$*, the function* $B : \mathrm{Int}(\mathcal{C}) \to \mathbb{R}$ *is called a* **reciprocal control barrier function** *(RCBF) defined for the set* $\mathcal{C}$*, if there exist* $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_{[0,a)}$ *such that for all* $x \in \mathrm{Int}(\mathcal{C})$*,*

$$\frac{1}{\alpha_1(h(x))} \leq B(x) \leq \frac{1}{\alpha_2(h(x))} \tag{14}$$
$$L_g B(x) = 0 \implies L_f B(x) < \alpha_3(h(x)). \tag{15}$$

Here $L_f B$, $L_g B$ are the Lie derivatives of B w.r.t. f, g. This definition is obtained from [12] and it is assumed that there are no restrictions on u. Similar to RCBF, we have the following definition of a zeroing control barrier function (ZCBF) [7].

Definition 2. *Given a set* $\mathcal{C} \subset \mathbb{R}^n$ *defined by* (11)-(13) *for a continuously differentiable function* $h : T\mathbb{Q} \to \mathbb{R}$*, the function* h *is called a* **zeroing control barrier function** *(ZCBF) defined for the set* $\mathcal{C}$*, if there exist* $\alpha \in \mathcal{K}_{[0,a)}$ *such that for all* $x \in \mathrm{Int}(\mathcal{C})$*,*

$$L_g h(x) = 0 \implies L_f h(x) > -\alpha(h(x)). \tag{16}$$

Note that the original definitions of RCBF and ZCBF in [7] do not use the strict inequality. Moreover, the ZCBF was defined for a larger set $\mathcal{D} \supset \mathcal{C}$, and with extended class $\mathcal{K}$ functions. In this paper, we are interested in safety of $\mathrm{Int}(\mathcal{C})$. We are reformulating with the view of ensuring continuity in the corresponding QP formulations. More details on the relationship between the QPs and the continuity requirements are provided in [12]. Given the RCBF and ZCBF, we know by [18, 4.19] that the following QP:

$$\begin{aligned} u^*(x) = \arg\min_u \quad & u^T u \qquad \text{(QP)} \\ \text{s.t.} \quad & L_f B(x) + L_g B(x) u \le \alpha_3(h(x)) \\ & \text{OR} \\ & L_f h(x) + L_g h(x) u \ge -\alpha(h(x)). \end{aligned}$$

not only yields a Lipschitz continuous control law, but also renders the set $\mathrm{Int}(\mathcal{C})$ forward invariant [7, Corollary 1]. The RCBF and the ZCBF each have their use cases in practical applications and will not be discussed here due to space limits (interested readers can refer to [7] for a detailed comparison).

C. Energy based CBFs

Given the robotic system described by (1) along with the associated properties, we know that the kinetic energy is

$$E(x) := \frac{1}{2}\dot{q}^T D(q)\dot{q}. \tag{17}$$

If we take the derivative of $E(x)$, we have

$$\begin{aligned} \dot{E}(x) &= \frac{1}{2}\dot{q}^T \dot{D}(q,\dot{q})\dot{q} + \dot{q}^T(-C(q,\dot{q})\dot{q} - G(q) + u) \\ &= \dot{q}^T(u - G(q)), \end{aligned} \tag{18}$$

where we have used Property 1. The above equation shows that the derivative of kinetic energy does not include D, C, which are dependent on the model. Further simplifications can be made on $G(q)$ by using the inequalities in Property 1. The main focus in this paper is to use this model-free formulation for construction of robust safety-critical control laws. To this end, we will now define a new class of RCBFs.

We assume that h is one of (7), (8), i.e., h is either $h_{i,p,\square}, i = 1, 2, \ldots, \square \in \{u, l\}$ or $h_{i,v,\square}, i = 1, 2, \ldots, \square \in \{u, l\}$. A more generic form with the constraints on task space (9) and (10) will be discussed in the next section.

We will study the position based constraint (7) first. Let $B_p : \mathrm{Int}(\mathcal{C}) \to \mathbb{R}$ be defined as

$$B_p(x) := \frac{1}{h(x)} + H(x).$$

where H denotes a differentiable function expressed in terms of the kinetic energy function $E(x)$ (17):

$$H(x) := \frac{E(x)}{1 + E(x)}. \tag{19}$$

By using (18), the derivative of B_p can be written as

$$\dot{B}_p(x, u) = -\frac{L_f h(x)}{h(x)^2} + \frac{\dot{q}^T(u - G(q))}{(1 + E(x))^2}, \tag{20}$$

and since $\dot{B}_p(x, u)$ satisfies (15), B_p is a valid RCBF. The resulting min-norm control law (QP) renders the set $\mathrm{Int}(\mathcal{C})$ safe (i.e., forward invariant).

Similar to the energy based RCBF (B_p), the energy based ZCBF is given by $z_p : T\mathbb{Q} \to \mathbb{R}$,

$$z_p(x) := -E(x) + \alpha_e h(x), \tag{21}$$

where $\alpha_e > 0$ is a constant. It can be verified that $z_p(x) \ge 0 \implies h(x) \ge E(x)/\alpha_e \ge 0$, which implies that the set defined by $z_p \ge 0$ is non-empty and is in $\mathcal{C}$:

$$\mathcal{C}_z := \{x \in T\mathbb{Q} : z_p(x) \ge 0\} \subset \mathcal{C}. \tag{22}$$

It can be verified that $z_p(x) > 0 \implies x \in \mathrm{Int}(\mathcal{C}_z)$, and safety of $\mathcal{C}_z$ (or $\mathrm{Int}(\mathcal{C}_z)$) implies $z_p(x(t)) > 0$ for all $t \ge 0$. More details about this class of ZCBFs are given in [9]. The derivative of (21) can be written as

$$\dot{z}_p(x, u) = -\dot{q}^T u + G^T\dot{q} + \alpha_e \dot{h}(x). \tag{23}$$

As mentioned earlier, both (20) and (23) do not require the knowledge of the Coriolis-centrifugal matrix C. Further modifications can be made using Properties 1 and 2 to obtain a model-free inequality. This is explained next.

Since our ultimate goal is to realize a model-free QP, we can replace $E(x), G(q)$ by their extreme values:

$$c_l|\dot{q}|^2 \le E(x) \le c_u|\dot{q}|^2, \quad -c_u|\dot{q}| \le \dot{q}^T G(q) \le c_u|\dot{q}|, \tag{24}$$

which are obtained by using Property 1. Based on the sign of $\dot{q}^T u$, appropriate bounds (lower or upper) must be substituted in (20). This results in two constraints similar to (QP) instead of one. We have the following two inequalities:

$$\psi_{00}(x) + \psi_{01}(x)u \le 0, \tag{25}$$

$$\psi_{10}(x) + \psi_{11}(x)u \le 0, \tag{26}$$

where

$$\begin{aligned} \psi_{00}(x) = \psi_{10}(x) &= -\frac{L_f h(x)}{h(x)^2} + \frac{|\dot{q}|c_u}{(1 + c_l|\dot{q}|^2)^2} - \alpha_3(h(x)), \\ \psi_{01}(x) = \frac{1}{(1 + c_l|\dot{q}|^2)^2}\dot{q}^T, &\psi_{11}(x) = \frac{1}{(1 + c_u|\dot{q}|^2)^2}\dot{q}^T. \end{aligned} \tag{27}$$

Accordingly, we define the point (state) to set mapping with these two constraints as

$$\mathbf{K}_{\mathrm{prcbf}}(x) := \left\{u \in \mathbb{R}^n : \ (25), (26) \text{ are true. } \right\}. \tag{28}$$

Similarly, we obtain the following inequality for ZCBF:

$$\phi_1(x) + \phi_2(x)u \ge 0, \tag{29}$$

where

$$\begin{aligned} \phi_1(x) &= \alpha_e \dot{h}(x) - c_u|\dot{q}| + \alpha(-c_u|\dot{q}|^2 + \alpha_e h(x)) \\ \phi_2(x) &= -\dot{q}^T, \end{aligned} \tag{30}$$

and the point to set mapping is obtained as

$$\mathbf{K}_{\mathrm{pzcbf}}(x) := \left\{u \in \mathbb{R}^n : \ (29) \text{ is true. } \right\}. \tag{31}$$

D. Energy based CBFs for velocity constraints

For the velocity constraints, we assume that h is of the type (8). Extension of these constraints for task space will be described in the next section. Depending upon the joint of interest, we can separate the dynamics (1) into two parts. If the goal is to constrain the velocity of the first joint, $\dot{q}_1$, then we have the following separation:

$$D_{11}\ddot{q}_1 + D_{12}\ddot{q}_b + C_1\dot{q} + G_1 = u_1, \tag{32}$$

$$D_{21}\ddot{q}_1 + D_{22}\ddot{q}_b + C_2\dot{q} + G_2 = u_b, \tag{33}$$

where q_b is the vector of remaining joint angles, and u_b is the vector of remaining control inputs. The matrices with subscripts, $D_\square, C_\square, G_\square$, are the block matrices of appropriate dimensions, which are self-explanatory. We can eliminate $\ddot{q}_b$ from (32) to yield

$$D_s(q)\ddot{q}_1 + C_s(q,\dot{q})\dot{q} + G_s(q) = u_1 - D_{12}(q)D_{22}^{-1}(q)u_b,$$

where $D_s(q)$ is the Schur complement form of the inertia matrix D ([16] Proposition 1):

$$D_s = D_{11} - D_{12}D_{22}^{-1}D_{21}, \tag{34}$$

which is symmetric and positive definite. C_s, G_s can similarly derived. Furthermore, by appropriate choice of c_l, c_u, D_s, C_s, G_s satisfy the same inequalities shown in Property 1 (see [16, Propositions 1 and 2] for a detailed description). With this setting, we choose the following RCBF:

$$B_v(x) = \frac{1}{h_v(x)}, \quad h_v(x) = \frac{1}{2}h(x)D_s(q)h(x).$$

Note that B_v is still a valid RCBF, since we can find $\alpha_1, \alpha_2 \in \mathcal{K}$ such that $\alpha_2(h(x)) \le h_v(x) \le \alpha_1(h(x))$ for $x \in \mathrm{Int}(\mathcal{C})$. Let $\psi(x) := -\hat{D}_{12}(q)\hat{D}_{22}^{-1}(q)$; where the symbol $\hat{}$ indicates the model estimate for D_{12}, D_{22}. Similarly, choose a tunable constant $0 < \gamma < 1$. Then we can use the following constraints in the (QP):

$$\frac{1}{\gamma}\psi_{20}(x) + \psi_{21}(x)[1 \;\; \gamma\psi(x)]u \le 0, \tag{35}$$

$$\frac{1}{\gamma}\psi_{30}(x) + \psi_{31}(x)[1 \;\; \gamma\psi(x)]u \le 0, \tag{36}$$

where

$$\psi_{20}(x) = \psi_{30}(x) = \frac{c_u}{{c_l}^2h^3}(\frac{1}{2}h|\dot{q}| + |\dot{q}|^2 + 1) - \alpha_3(h(x)),$$

$$\psi_{21}(x) = \frac{1}{{c_l}^2h^3}, \qquad \psi_{31}(x) = \frac{1}{{c_u}^2h^3}, \tag{37}$$

Similarly for ZCBF we get the following inequality:

$$\frac{1}{\gamma}\phi_3(x) + \phi_4(x)[1 \;\; \gamma\psi(x)]u \le 0, \tag{38}$$

where

$$\phi_3(x) = c_u h(\frac{1}{2}h|\dot{q}| + |\dot{q}|^2 + 1) - \alpha(h(x)),$$

$$\phi_4(x) = h. \tag{39}$$

Having obtained the model-free inequalities, we have the following results which were established in [8], [9].

Lemma 1. *Given the position based constraint function h and the corresponding set $\mathcal{C}$ defined by* (11) - (13) *then the following QP:*

$$u^*(x) = \arg\min_u \quad u^T u \qquad \text{s.t.} \quad (25), (26) \quad \text{OR} \quad (29) \tag{40}$$

guarantees safety (i.e., forward invariance of $\mathrm{Int}(\mathcal{C})$ *for RCBF, and forward invariance of* $\mathrm{Int}(\mathcal{C}_z)$ *for ZCBF).*

Lemma 2. *Given the velocity based constraint function h and the corresponding set $\mathcal{C}$ defined by* (11) - (13) *then, for a small enough $\gamma > 0$, the following QP:*

$$u^*(x) = \arg\min_u \quad u^T u \qquad \text{s.t.} \quad (35), (36) \quad \text{OR} \quad (38) \tag{41}$$

guarantees safety i.e., forward invariance of $\mathrm{Int}(\mathcal{C})$.

Proofs of Lemmas 1 and 2 can be found in [8], [9]. With these results, we now extend our work for task space safety-critical control.

IV. Control barrier functions for task space

The barrier functions defined for joint space can be reformulated for task space with the help of the Jacobian $J(q)$ which maps the control u to the task space dynamics. Since we know from Property 2 that $c_l \le \|D_e(q)\| \le c_u$ and $c_e|\dot{q}| \le |\dot{e}|$, for some constant c_e. Thus we have:

$$\frac{1}{1 + \dot{e}^T D_e \dot{e}} \le \frac{1}{1 + c_l|\dot{e}|^2} \le \frac{1}{1 + c_l c_e^2|\dot{q}|^2}. \tag{42}$$

We can replace $c_l' = \min\{c_l, c_e^2\}$ to yield final inequality. The upper bound c_u' can be similarly obtained. Accordingly, (25) and (26) can be reformulated for the task-space, where $\psi_{00}, \psi_{01}, \psi_{10}, \psi_{11}$ are replaced with $\psi_{00}', \psi_{01}', \psi_{10}', \psi_{11}'$ respectively:

$$\psi_{00}'(x) + \psi_{01}'(x)u \le 0, \tag{43}$$

$$\psi_{10}'(x) + \psi_{11}'(x)u \le 0, \tag{44}$$

with

$$\psi_{00}'(e) = \psi_{10}'(e) = -\frac{L_f h(e)}{h(e)^2} + \frac{|\dot{q}|c_u}{(1 + c_l'|\dot{q}|^2)^2} - \alpha_3(h(e)),$$

$$\psi_{01}'(e) = \frac{1}{(1 + c_l'|\dot{q}|^2)^2}\dot{e}^T J^{-T} = \frac{1}{(1 + c_l'|\dot{q}|^2)^2}\dot{q}^T,$$

$$\psi_{11}'(e) = \frac{1}{(1 + c_u'|\dot{q}|^2)^2}\dot{e}^T J^{-T} = \frac{1}{(1 + c_u'|\dot{q}|^2)^2}\dot{q}^T. \tag{45}$$

With this substitution, it can be verified that the resulting ψ_{01}', ψ_{11}' differ from ψ_{01}, ψ_{11} only by a scaling factor. This is very important for including multiple constraints.

For ZCBF based constraint for task space, we can obtain the following in a similar manner:

$$\phi_1'(e) + \phi_2'(e)u \ge 0, \tag{46}$$

which is similar to (29), and the new terms ϕ'_1, ϕ'_2 are obtained accordingly. Further, for the end effector velocity constraints, we have

$$\frac{1}{\gamma}\psi'_{20}(e) + \psi'_{21}(e)[1\ \gamma\psi'(e)]J^{-\top}(q)u \leq 0, \tag{47}$$

$$\frac{1}{\gamma}\psi'_{30}(e) + \psi'_{31}(e)[1\ \gamma\psi'(e)]J^{-\top}(q)u \leq 0, \tag{48}$$

where the ψ's above are obtained are accordingly. Similarly for ZCBF, (38) can be reformulated as

$$\frac{1}{\gamma}\phi'_3(e) + \phi'_4(e)[1\ \gamma\psi'(e)]J^{-\top}(q)u \leq 0, \tag{49}$$

where ϕ'_3 and ϕ'_4 are obtained accordingly.

With the new constraints, (43), (44), (46) and the rest, reformulated versions of Lemmas 1, 2 can be established. Due to space constraints, stating this result will be omitted. We will now incorporate tracking along with the CBF based constraints and present our main results.

V. Unification of Tracking and Safety

A. *Control Lyapunov Function for Task space*

In order to stabilize the manipulator to a desired position and orientation in the task space, we employ QP based PD control law which is independent of any model parameters as given in [19]. Asymptotic convergence cannot be guaranteed, but the error can be reduced to an arbitrarily small bound subject to suitable tuning of the gains. The cost function is chosen such that it minimizes the difference between control torque u and a reference torque u_{ref} given by:

$$\min_{u,\delta}(u - u_{ref}(t,x))^\top(u - u_{ref}(t,x)) + \delta^2.$$

$$\text{s.t. } (\beta(e - e_d)^\top + (\dot{e} - \dot{e_d})^\top)(K_p(e - e_d) + K_d(\dot{e} - \dot{e_d}) + (\beta(e - e_d)^\top + (\dot{e} - \dot{e_d})^\top)J^{-\top}u \leq \delta \tag{50}$$

where $\beta = \frac{k_0}{1+|e-e_d|}$, K_p, K_d are the gain matrices, e_d is the desired position in task space. Here the constant k_0 is chosen such that it satisfies $k_0 \leq \frac{\sqrt{||K_p||||D_e||}}{||D_e||}$. We have the following theorems.

Theorem 1. *Given the set of position and velocity constraints for a manipulator* (7)-(10)*, and the corresponding reciprocal control barrier functions, the QP of the form*

$$\begin{aligned} \min_{u,\delta}\quad & (u - u_{ref}(t,x))^\top(u - u_{ref}(t,x)) + \delta^2 \\ \text{s.t.}\quad & (50) \\ & (25), (26), \text{with } h = h_{i,p,\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \\ & (35), (36), \text{with } h = h_{i,v,\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \\ & (43), (44), \text{with } h = h_{i,e,\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \\ & (47), (48), \text{with } h = h_{i,\dot{e},\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \end{aligned} \tag{51}$$

guarantees safety i.e., forward invariance of intersection of all sets ($\mathrm{Int}(\mathcal{C})$*'s) defined for each* h.

Proof. Due to space constraints, a rough sketch of the proof will be provided. We will be mainly using the results from [8, Theorems 1,2] and [7, Theorems 1,2,3]. We will first establish the feasibility of the QP shown above. By virtue of their property, each of the constraints can be represented as

$$a_k + b_k \begin{bmatrix}\delta \\ u\end{bmatrix} \leq 0, \tag{52}$$

where a_k, b_k are indexed by k based on the constraint. For the CLF and the velocity constraints, b_k is a non-zero row vector for all $x \in \mathrm{Int}(\mathcal{C})$. For all other constraints, while a_k changes based on the constraint function h, b_k is always of the form $s * \dot{q}^T$, where $s > 0$ is some scaling factor (see (27) and (45) for comparison). Hence, by re-scaling all of these constraints, i.e., by dividing each position based safety constraint by the scaling factor, we will have identical $b's$. Therefore, we can replace all of these constraints by a single constraint

$$\max\{a_1, a_2, \dots\} + \dot{q}^T u \leq 0, \tag{53}$$

and since the $\max$ function is continuous, the original QP can be replaced with a QP having a single position based constraint along with others. Finally, by using the results from [12, Theorem 1], we can guarantee feasibility and continuity, thereby guaranteeing safety. □

Theorem 2. *Given the set of position and velocity constraints for a manipulator* (7)-(10)*, and the corresponding zeroing control barrier functions, the QP of the form*

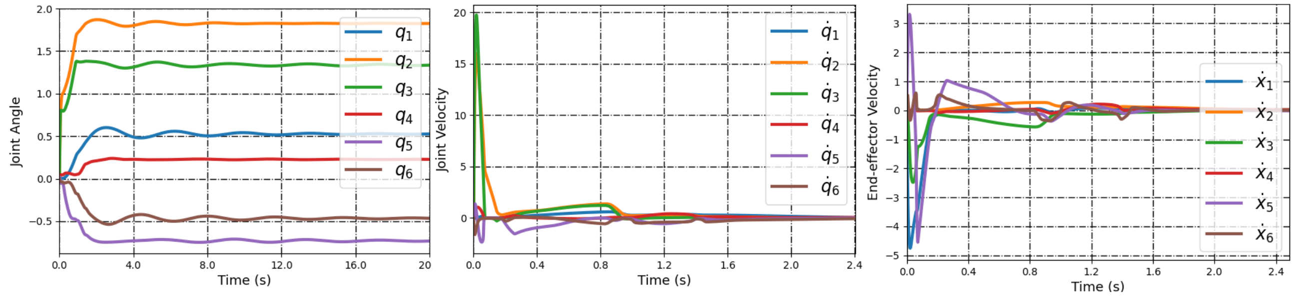
$$\begin{aligned} \min_{u,\delta}\quad & (u - u_{ref}(t,x))^\top(u - u_{ref}(t,x)) + \delta^2 \\ \text{s.t.}\quad & (50) \\ & (29), \text{with } h = h_{i,p,\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \\ & (38), \text{with } h = h_{i,v,\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \\ & (46), \text{with } h = h_{i,e,\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \\ & (49), \text{with } h = h_{i,\dot{e},\Box}, i \in \{1,2,\dots\}, \Box \in \{u,l\} \end{aligned} \tag{54}$$

guarantees safety i.e., forward invariance of intersection of all sets ($\mathrm{Int}(\mathcal{C})$*'s or* $\mathrm{Int}(\mathcal{C}_z)$*'s) defined for each* h.

Proof of Theorem 2 will be similar to Theorem 1 and hence will be omitted.

VI. Results

The Kinova Jaco2 6 DOF robotic arm is used for performing simulations inside the MuJoCo simulation environment. It consists of six hinge joints which constitute the configuration $q = \{q_1, q_2, q_3, q_4, q_5, q_6\}$ of the arm. It also consists of an end effector with 3 fingers each having two hinge joints for performing pick and place operations (as shown in Fig. 1). Table I displays the parameters used for performing simulation such that the end effector of the robot manipulator safely reaches the desired configuration $\{x, y, z, roll, pitch, yaw\} = \{-0.4, 0.3, 0.6, 0.1, 0.1, 0.1\}$. Note that we have successful tracking for multiple desired configurations and the plots shown in the paper are for a specific desired value. The simulations are performed for reciprocal CBFs (51), and it can be seen from Fig. 2 and Fig. 3 that the manipulator reaches the desired configuration in



(a) (b) (c)

Fig. 2: Response curves for (a) joint angles (b) joint velocities and (c) end-effector linear and angular velocities vs time.

TABLE I: Simulation Parameters

Parameters	Value	
	RCBF	ZCBF
$K_p, K_d(u_{ref})$	$20, 2$	$8000, 2$
α_e, α	-	$5, 0.2$
α_3	1000	-
K_p, K_d (CLF constraints)	$400, 100$	$40, 5$
q_{max}, q_{min}	$2\pi/3,\ -2\pi/3$	
$q_{d,max}, q_{d,min}$	$21,\ -21$	
x_{max}, x_{min}	$1.2,\ -1.2$	
$x_{d,max}, x_{d,min}$	$8,\ -8$	
c_l, c_u	2 , 20	
τ_{max}, τ_{min}	$100,\ -100$	
k_0	0.01	
γ	0.007	

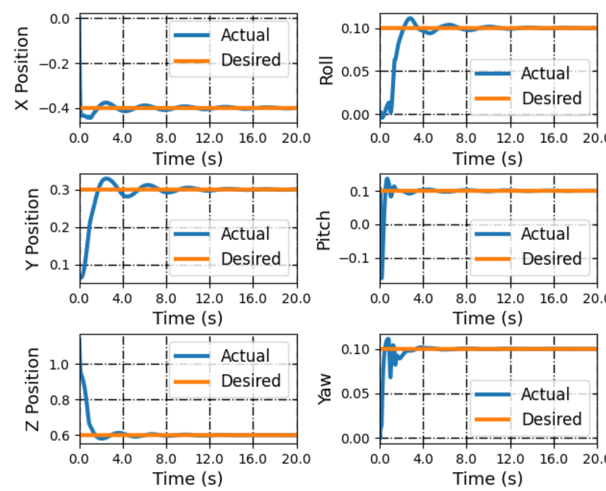


Fig. 3: Response curves for desired and actual end-effector position and orientation vs time.

15 seconds while satisfying the end effector safety limits i.e. $(-1.2, 1.2)$. Fig. 2 (a) depicts the variation of joint angle with time and it can be seen that the all the six joint angles remain within the safety limits $(-2\pi/3, 2\pi/3)$ rad. Fig. 2 (b) and 2 (c) show the variation of joint and end-effector linear and angular velocities with time and it can be seen that that joint velocities and end-effector velocities remain within the safety limits i.e. $(-21, 21)$ and $(-8, 8)$ respectively. Similarly the control torque for each joint remains continuous and within the limits $(-100, 100)$ Nm as shown in Fig. 4.

The simulation results for the case of zeroing CBFs (54) are shown in Fig. 5, 6, 7, 8 and 9 depicting the variation of end-effector position and orientation, joint angles, joint velocities, end-effector velocities and control torque respectively with time. The same set of safety constraints were used for ZCBF as used for RCBF as shown in Table I. From Fig. 5, it can be observed that the manipulator reaches the desired configuration within 20 seconds while satisfying the different constraints. Similarly, the torques shown in Fig. 9 are continuous and remain with the limits $(-100, 100)$ Nm.

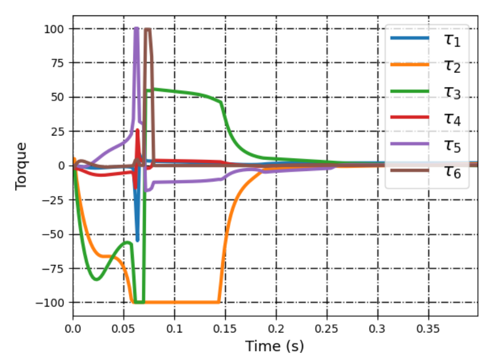


Fig. 4: Control torque components vs time.

VII. Conclusion

In this paper, we demonstrate multi-objective QPs for robotic manipulators via energy based CBFs. Practical manipulators industries and other locations are frequently operated in constrained environments. Hence, safety critical control becomes very important for tasks that involve active interactions between the robot and humans. The energy based CBFs provide an effective tool for addressing the problem of safety when the number of constraints are large. We provide feasibility, continuity and safety guarantees with the proposed formulation. Future work, will involve extending this work for a broader class of robotic systems.

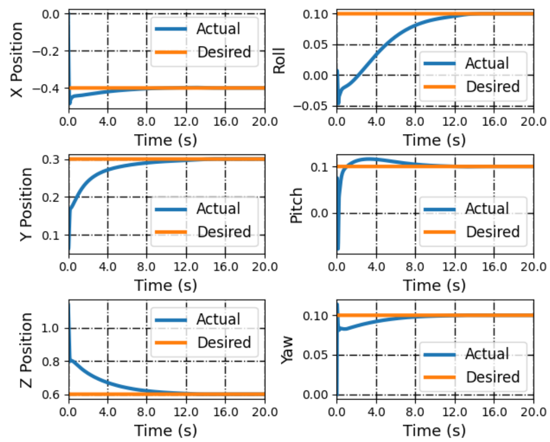


Fig. 5: Response curves for desired and actual end-effector position and orientation vs time.

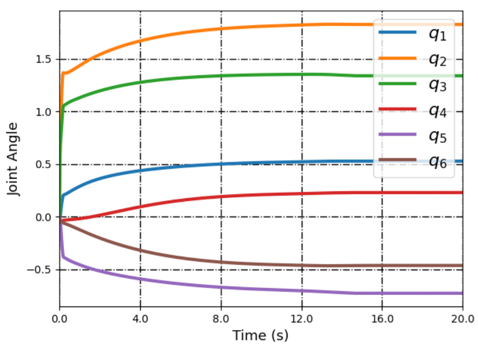


Fig. 6: Response curves for joint angles vs time.

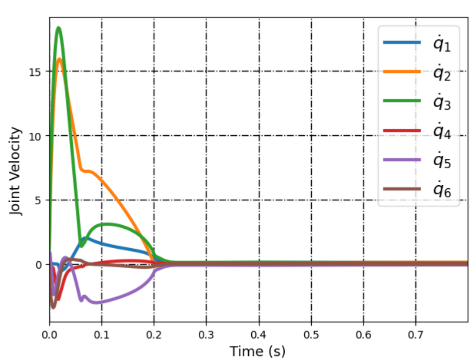


Fig. 7: Response curves for joint velocities vs time.

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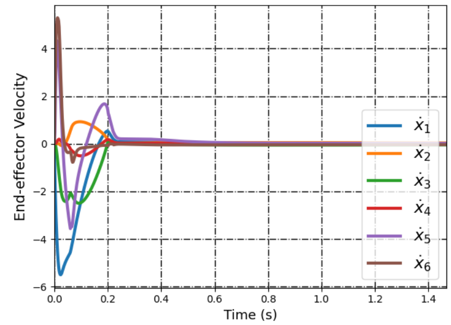

Fig. 8: Response curves for end-effector linear and angular velocity components vs time.

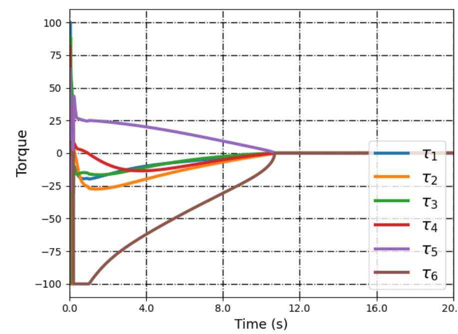

Fig. 9: Control torque components vs time.

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